RESEARCH DEPARTMENT

Methods for measuring the screening factor and characteristic impedance of systems of parallel conductors

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METHODS FOR MEASURING THE SCREENING FACTOR AND CHARACTERISTIC IMPEDANCE OF SYSTEMS OF PARALLEL CONDUCTORS

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SUMMARY

Three methods are described for determining the characteristic parameters of systems of three conductors. These methods have been applied to the design of cage-driven medium-frequency aerials, and also to the testing of symmetrical directional couplers. In each case their accuracy has been assessed by measurements on systems whose parameters could be calculated.

1. INTRODUCTION

In the design of cage-driven medium-frequency (m.f.) aerials, i.e. an earthed mast surrounded by a set of parallel-connected conductors, the parameters of interest are the screening factor and characteristic impedance of the two-conductor system.[1] In order to measure these parameters it is convenient to use a small-scale model and to introduce a third conductor which is sufficiently distant from the model, so as not to invalidate the results for the two-conductor system. The additional conductor may take the form of a large uniform cylindrical screen which encloses the structure under test.

Three experimental methods are described which may be used to determine the parameters which define the behaviour of such systems. The procedure requires only three independent capacitance measurements, and these are sufficient for determining any TEM-mode performance of the structure under test. The accuracy of the methods was assessed by measurements on systems whose parameters could be calculated.

The methods described can be applied not only in the design of cage-driven m.f. aerials, but also to the testing of twin-mode structures, such as symmetrical directional couplers, which depend for their operation on the transmission of 'push-push' and 'push-pull' modes within a three-conductor system.[2]

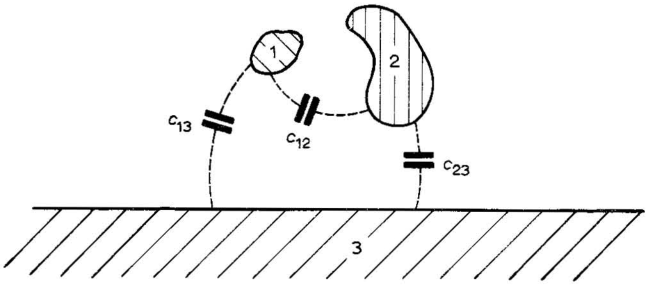

Fig. 1 - The cross-section of a 3-conductor system

2. METHOD OF MEASUREMENT

If we consider the three-conductor line system of Fig. 1, the partial capacitances per unit length c_{12}, c_{23} and c_{13} of the equivalent π-circuit provide all the data required to specify the transmission properties of such a system, providing it is uniform and immersed in an infinite, homogeneous and lossless material.[*]

Suppose the conductors 1 and 2, in general of dissimilar cross-section, are connected in parallel, and given an electrostatic charge; the charge ratio, q_1/q_2, is by definition equal to the ratio of the partial capacitances c_{13}/c_{23}. In the a.c. case, since current is proportional to rate of charge with time, the current ratio $k = i_1/i_2$ is also equal to c_{13}/c_{23}.[†] If conductor 3 is removed to a great distance the ratio c_{13}/c_{23} tends to a limiting value which may be regarded as the 'free space' screening ratio.

Another way of describing the screening performance is to define a ratio s, the screening factor,[1] as:

$$s = i_2/(i_1 + i_2)$$

so that

$$s = c_{23}/(c_{13} + c_{23}) = 1/(1 + k) \tag{1}$$

The advantage of this definition is that the screening factor s now varies between 0 and 1, $s = 1$ representing perfect screening. The characteristic impedance Z_0 of the transmission line formed by conductors 1 and 2 alone is given by $Z_0 = 1/vc_{12}$ where v is the phase velocity. Hence low frequency capacitance measurements determining c_{13}, c_{23} and c_{12} will yield k, s and Z_0.

Suppose now that conductors 1 and 2 are identical in form with $c_{13} = c_{23}$; if they are driven in parallel relative to conductor 3 the system becomes 'unbalanced' (i.e. behaves in the push-push mode) and the characteristic admittance of the system is given by $Y_u = 2vc_{23}$. If the same conductors are driven from a generator balanced about the third conductor, the system becomes 'balanced' (i.e. operates in the push-pull mode) with a characteristic admittance $Y_b = v(c_{12} + \frac{1}{2}c_{23})$. If such a structure is used as a symmetrical directional coupler then the coupler admittance[2] is given by:

$$Y_0^2 = Y_u Y_b \tag{2}$$

and the output ratio K of a quarter-wave coupler is given by $K = \frac{1}{2}(r - 1/r)$ where:

$$r = 2\sqrt{(Y_b/Y_u)} \tag{3}$$

Hence low-frequency capacitance measurement of c_{12} and c_{23} determines both the output ratio K and the coupler admittance Y_0.

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[*] This includes free space as a particular case.

[†] It may also be shown that if equal and opposite charges are placed on conductors 1 and 2, or if equal and opposite currents flow, then $-v_2/v_1$ is also equal to k. Use is sometimes made of this property when calculating the performance of systems containing parallel conductors.

The most convenient measurements to make on such systems are as follows:

(a) Capacitance per unit length between 1 and the parallel combination of 2 and 3, : c_a

(b) Capacitance per unit length between 2 and the parallel combination of 3 and 1, : c_b

(c) Capacitance per unit length between 3 and the parallel combination of 1 and 2, : c_c

We then have:

$$\begin{aligned} c_a &= c_{12} + c_{13} \\ c_b &= c_{12} + c_{23} \\ c_c &= c_{13} + c_{23} \end{aligned} \tag{4}$$

Equations (4) yield c_{12}, c_{13}, c_{23} in terms of measured quantities c_a, c_b, c_c thus:

$$\begin{aligned} c_{12} &= \tfrac{1}{2}(c_a + c_b - c_c) \\ c_{23} &= \tfrac{1}{2}(c_b + c_c - c_a) \\ c_{13} &= \tfrac{1}{2}(c_a + c_c - c_b) \end{aligned} \tag{5}$$

As a check, we may also measure between two conductors with the third one 'floating'; for example, the capacitance between 2 and 3 in the presence of 1 becomes $c_{23} + c_{12}c_{13}/(c_{12} + c_{13})$; alternatively, with some bridges, c_{12} may be determined directly with conductor 3 earthed.[3] For structures in which conductor 1 is well screened by conductor 2, c_{12} may be measured directly using an unbalanced bridge with little error if the outer conductor is earthed, since the third conductor then has an insignificant effect on the measurement.

In terms of the measured line capacitances c_a, c_b and c_c, the system parameters become:

Unbalanced admittance $\quad Y_u = vc_c \quad$ (6)

Balanced admittance $\quad Y_b = v(c_a - c_c/4) \quad$ (7)

Screening factor $\quad s = (c_c + c_b - c_a)/2c_c \quad$ (8)

Impedance of two-conductor system $\quad Z_0 = 2/v(c_a + c_b - c_c) \quad$ (9)

3. EXPERIMENTAL METHODS

Direct capacitance measurements in order to determine the parameters of systems of conductors of finite length are liable to error because the electric field is distorted at the ends of the conductors. Three methods which enable the errors due to these end effects to be overcome are described in this section; two involve the measurement of capacitance directly, the third makes use of a resistance analogue.

3.1. Double Measurement Technique

In this method, the end effects are determined by measuring two different lengths of the same structure whilst at the same time ensuring that the end conditions for the two lengths are identical.

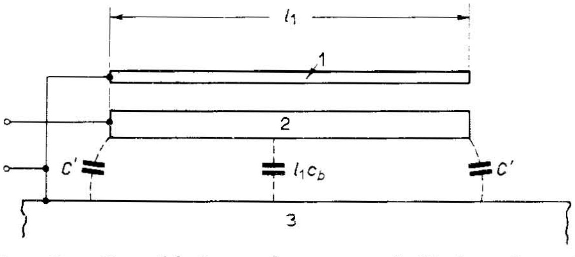


Fig. 2 - Parallel conductors of finite length, showing additional capacitances due to end effects. The conductors are connected for the measurement of c_b

Suppose c_b is measured over two lengths l_1 and l_2, the shorter of the two lengths, l_1, being made sufficiently large that, if it is increased, the field in the extra length at the centre is substantially uniform. Referring to Fig. 2, the quantities measured are:

$$\begin{aligned} C_1 &= l_1c_b + 2C' \\ C_2 &= l_2c_b + 2C' \end{aligned} \tag{10}$$

where c_b is the capacitance per unit length of an infinite line and C' the additional capacitance at each end due to fringing. We therefore have:

$$c_b = (C_2 - C_1)/(l_2 - l_1) \tag{11}$$

The method is accurate if the end conditions are identical for each measurement; this is especially important if the fringe field represents a large fraction of the total length. It is not necessary to achieve identical conditions at each end since equation (11) will still hold. It is sometimes convenient to allow the third conductor to overlap considerably at one or both ends, so that for the second measurement only two of the conductors need be adjusted. Satisfactory results will normally be obtained if l_2 is made approximately equal to twice l_1 and l_1 is made at least ten times the overall width of conductors 1 and 2 taken together.

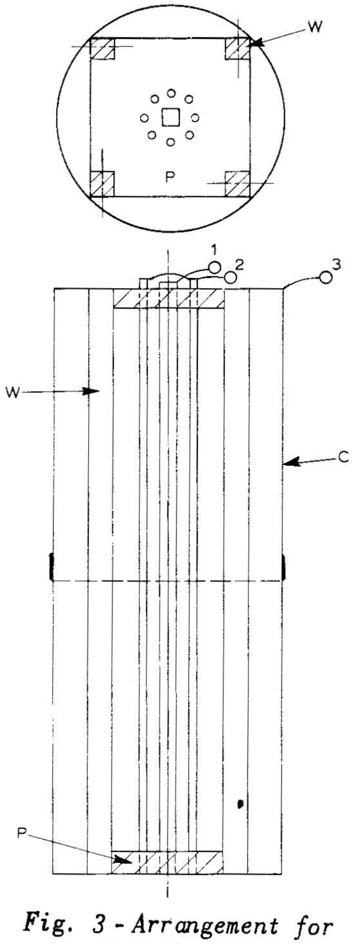


Fig. 3 - Arrangement for measuring the screening factor of a cage-driven aerial structure

For the measurement of the screening factor between two conductors 1 and 2 it is convenient to enclose the system in a third conductor, 3, whose inner dimension is much larger than the overall dimension of 1 and 2 taken together; the value of s thus determined is then an accurate measure of the 'free space' screening factor. In order to measure the screening factor of a cage-driven aerial in small-scale model form, an arrangement such as shown in Fig. 3 may be used. The structure representing the mast-inner surrounded by a cage of stretched wires is supported by two perspex end plates P, held at the ends of four wooden posts, W; the outer cylinder C is in two lengths to facilitate the double measurement. The end plates are included in the end effects and are eliminated in measurement as previously described. Using a cylinder 7½ inch (19 cm) diameter and 6 ft (1·8 m) long with internal structures of cross-sectional dimension up to one-tenth cylinder diameter, the measured screening factors ignoring end effects were within 3% of the calculated values for systems whose parameters could be derived theoretically. Using the double measurement technique, the accuracy varied from under 1% error with the cage/mast structure occupying one-twentieth of the outer cylinder diameter, to approximately 3% error with the cage dimension doubled. The measurements were performed on calculable arrangements with s greater than 0·8.

The double measurement technique has also been used to design a hybrid structure which was required for a negative-impedance amplifier. This hybrid was a directional coupler (70 ohms, 3 dB, 200 Mc/s) consisting of a trough outer (conductor 3) and quarter-wave twin strips as coupling line. A rough design using theoretical approximations was first produced, and the inner conductors were then adjusted in width and spacing until the correct values of Y_u and Y_b, given by equations (6) and (7), were achieved. Such a technique, requiring successive adjustment and measurement, is facilitated if adjustment of one parameter, for example strip spacing, produces a very much greater change in one partial capacitance (c_{12}) than it does in the others (c_{13}, c_{23}). The accuracy of the method was such that a directivity of 36 dB was achieved in the final coupler after three adjustments.

3.2. The 'Guard-Ring' Method

Another method of eliminating end-effects is to restrict measurement to a portion of the system in which the field is uniform. A technique for achieving this is illustrated in Fig. 4, which shows how guard extensions, G, may be employed in a two-conductor measurement. If one side of the detector is earthed, and the guards are also placed at earthed potential, the condition of bridge balance also brings conductor A to earth potential. The guards then act as a mere extension of A producing uniform field between A and B; this is the condition required. The principle may be extended to a 3-conductor system, the arrangements for the three capacitance measurements being shown in Fig. 5. The two inner conductors are each split into three lengths insulated from one another with the narrowest practicable gaps, and the lead-out wires from the central lengths are made very fine and run, either parallel and close to the inner conductors, or within them if they are tubular.

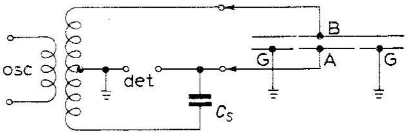
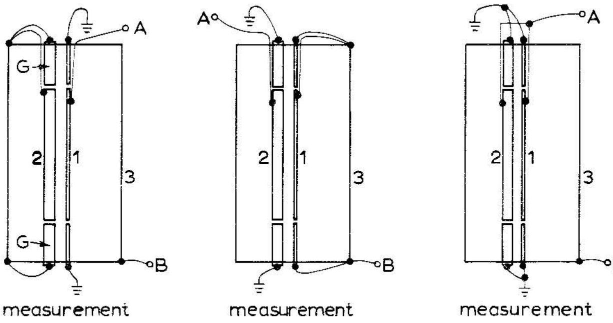


Fig. 4 - Bridge measurement using the guard-ring method



measurement of C_a

measurement of C_b

measurement of C_c

Fig. 5 - Application of the guard-ring method of measurement to 3-conductor systems

Measurements on simple structures of metal tubes placed within a large cylindrical cage have produced screening factors within 1% of the theoretical values. It is, however, difficult to apply this method to a complicated structure such as a cage of wires surrounding an inner conductor because the lead-out wires from the central part of the cage are comparable in size with the guard wires and distort the field in the guard extensions; this leads to considerable errors.

3.3. Resistance Analogue Method

Resistance paper with uniform resistance per square may be used as an analogue for two-dimensional electrostatic problems. The profiles of the cross-sections of the conductors are painted on the paper and connected to electrodes across which a d.c. voltage is applied. The lines of current flowing across the paper correspond to electrostatic lines of force, and equipotentials are the same as in the electrostatic case; thus the conductance between two electrodes is proportional to capacitance per unit length.

Teledeltos paper* is one such analogue paper which consists of a resistive sheet covered by a thin insulating tissue. The conducting profiles are painted on the tissue side of the paper with a silver solution whose solvent pierces the insulating layer and, on subsequent drying, bonds the silver on to the resistive sheet. In order to ensure that the silver profile is a perfect equipotential, it is advisable to pin the profile every few centimetres, connecting the pins by tinned copper wire, laying flat on the paper surface. The tinned copper wire is then immersed in the silver coating, and the conducting wire is soldered on to the pins which may conveniently be used as take-off points for measurement. A typical layout for a 3-conductor problem is shown in Fig. 6.

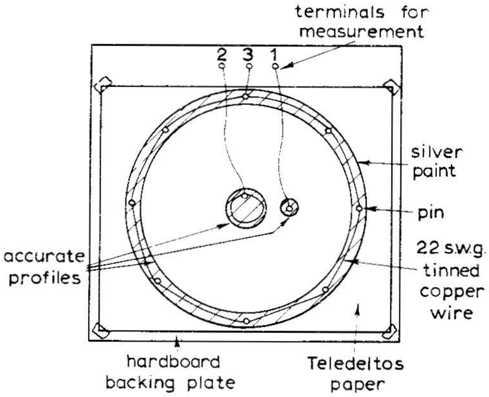


Fig. 6 - A typical layout for the resistance paper analogue method

The measurement procedure involves two steps. Firstly, the portion of paper to be used is calibrated by measuring the resistance between two conducting profiles of a system of known capacitance, and for this a simple coaxial structure is most convenient. Secondly, another part of the same piece of paper is used to map and measure the required unknown system. It should be noted that Teledeltos papers exhibit an anisotropic resistance error due to grain developed in manufacture; for coaxial structures with near radial flux flow this error can be neglected. For certain other shapes of conductors it is advisable to take the mean of two measurements made in orthogonal directions.

For small and complicated profiles, such as a cage of wires, it is more accurate to map the conducting system using a stencil. It has been found that a perspex plate suitably shaped and backed with a thin film of MS4 silica gel or vaseline will act as a practical mask. With this method it is preferable to insert the conducting pins before the stencil is put in place and the silver solution introduced.

For assessments of absolute capacitance it is essential to calibrate the paper by carrying out measurements on calculable systems, and also to check the uniformity by making a number of such measurements. For the assessment of relative capacitance it is necessary only to check the uniformity. One sample of paper gave results within 1% of the theoretical.

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*This is the trade name of resistance paper manufactured by Servomex Controls Ltd.

For the measurement of the screening factor of a cage of wires, this method is severely limited because it is difficult to scale the cage wires accurately when mapping them on to the resistance paper; in fact a system comprising an eight-wire cage round a cylindrical mast yielded an accuracy not better than 10%, which was unacceptable for the application in view.

4. CONCLUSIONS

Methods have been described for measuring the important parameters of systems of parallel conductors, in particular the screening factor.

For most problems, the double measurement technique is normally sufficiently accurate and versatile. The 'guard-ring' method is much more elegant and yields excellent accuracy, but becomes mechanically difficult if the structure is at all complicated.

Resistance paper used as a capacitance analogue avoids difficulties due to end effects but does not produce sufficiently accurate results for systems containing conductors of small cross section. It has the advantage, however, that an approximate measurement of the properties of a complicated structure can be quickly made and used as a basis for further design, measurement by one of the other techniques being subsequently used.

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